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Relativistic Calculations of Angular Correlations of Particles Issued from an Interaction-Free Resonance Decay

1. Introduction

While results on relativistic transformations of an isotropic angular distribution are well-known [1, 2], results about the angular correlations between the particles of an interaction-free particle decay are less well-known. We solve this problem by the jacobian method [1], which is easy to handle even for rather complicated problems. We consider decays into two particles with equal and different masses. The resulting formulae can be easily programmed to be used as subroutines in a study of particle reactions.

In an external report [3], we presented a more complete version of this work with calculation flowcharts and FORTRAN programs. This report is available on request.

2. Angular Correlation Between Two Particles Issued from a Resonance Decay

The notations adopted in this paper are those of Ref. [1].

2.1. *Decay into Two Equal Mass Particles*

Let K be the system in which the resonance is emitted and K' the system bound to this resonance. The decay is isotropic in K'. The polar coordinates of particle i are $\mathbf{p}_i(p_i, \theta_i, \varphi_i)$, but in order to study angular correlations we use relative polar angles θ_{12} and φ_{12}. We pass from (θ_1, φ_1) to $(\theta_{12}, \varphi_{12})$ by a rotation.

By using Jacobians, we can write:

$$\frac{d^3\sigma}{dE_1\, d\cos\theta_{12}\, d\varphi_{12}} = \frac{d^2\sigma'(E_1', \Omega_1')}{dE_1'\, d\Omega_1'} \frac{\partial(E_1', \Omega_1')}{\partial(E_1, \Omega_1)} \frac{\partial(E, \cos\theta_1, \varphi_1)}{\partial(E, \cos\theta_{12}, \varphi_{12})}, \tag{1}$$

where E_i and E_i' are the energies of particle i in K and K' systems.

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Thus we obtain, for the wanted angular correlation in the K system:

$$\frac{d\sigma}{d(\cos\theta_{12})} = \frac{1}{2p_1'} \int_{m_1}^{\infty} p_1 \delta(E_1' - E_0')\, dE_1$$

and

$$\frac{d\sigma}{d(\cos\theta_{12})} = \frac{1}{2p_1'} \sum_{\alpha} | p_1 (dE_1/dE_1') |_{E_1 = E_1^{(\alpha)}}\,; \tag{2}$$

in which E_0' is the energy of the particle in the K' system and where $E_1^{(\alpha)}$ are the E_1 roots of the equation $E_1' - E_0' = 0$.

In the K' system, we have

$$p_1' = p_2' \qquad \text{and} \qquad \theta_{12}' = \theta_1' + \theta_2' = \pi. \tag{3}$$

By using relativistic transformations, we get the relations

$$\begin{aligned} p_1^2 + p_2^2 + 2p_1p_2\cos\theta_{12} &= 4\beta^2\gamma^2E_1'^2, \\ E_1 + E_2 &= 2\gamma E_1', \\ E_R = \gamma E_R' &= 2\gamma E_1', \end{aligned} \tag{4}$$

where E_R is the energy of the resonance.

From these relations, we can derive dE_1/dE_1', so that the angular correlation (2) can now be written:

$$\frac{d\sigma}{d\cos\theta_{12}} = \frac{1}{2p_1'} \sum_{\alpha} \left[p_1^2 \left| \frac{4\beta^2\gamma^2E_1'p_2 - 2\gamma E_2(p_2 + p_1\cos\theta_{12})}{p_1p_2(E_1 - E_2) + \cos\theta_{12}(E_1p_2^2 - E_2p_1^2)} \right| \right]_{E_1 = E_1^{(\alpha)}}. \tag{5}$$

We have now to find $E_1^{(\alpha)}$. By eliminating E_2 between the relations (4), we obtain

$$(E_1^2 - 2\gamma E_1E_1' - m^2) + 2E_1'^2 = -\cos\theta_{12}[(E_1^2 - 2\gamma E_1E_1' - m^2)^2 - 4m^2\gamma^2E_1'^2]^{1/2}, \tag{6}$$

where m is the mass of the emitted particles. Setting

$$x = E_1^2 - 2\gamma E_1E_1' - m^2, \tag{7}$$

relation (6) can be written

$$x + 2E_1'^2 = -\cos\theta_{12}[x^2 - 4m^2\gamma^2E_1'^2]^{1/2}; \tag{8}$$

the roots of equation (8) are

$$x_{1,2} = [-2E_1'^2 + 2\epsilon E_1'\cos\theta_{12}(E_1'^2 - m^2\gamma^2\sin^2\theta_{12})^{1/2}]/\sin^2\theta_{12}\,, \tag{9}$$

with $\epsilon = +1$ for x_1 and $\epsilon = -1$ for x_2.

The corresponding values for $E_1^{(\alpha)}$ are

$$E_1^{(\alpha)} = \gamma E_1' + \epsilon[\gamma^2 E_1'^2 + m^2 + x_i]; \tag{10}$$

$\alpha = 1, 2$ with $i = 1$ and $\epsilon = \pm 1$, and $\alpha = 3, 4$ with $i = 2$ and $\epsilon = \pm 1$.

The discussion of the different equations gives the following results:

— if $E_1' \geqslant m\gamma$ and if: $-1 < \cos\theta_{12} < C_2$

with $C_2 = [E_1'^2(\gamma^2 - 2) + m^2]/(\gamma^2 E_1'^2 - m^2)$ we find that only the two roots E_1 [3, 4] are suitable.

— if $E_1' < m\gamma$ we have to distinguish two possibilities:

(1) If $E_1'^2 \geqslant m^2(1 + \beta^2)$ we find that if $C_1 \leqslant \cos\theta_{12} \leqslant C_2$, the four roots are suitable,

$$\text{with:} \quad C_1 = (m^2\gamma^2 - E_1'^2)^{1/2}/m\gamma;$$

and if: $C_2 < \cos\theta_{12} \leqslant 1$, only the two roots E_1 [2, 3] are suitable.

(2) If $E_1'^2 < m^2(1 + \beta^2)$ then we have $| C_2 | \leqslant \cos\theta_{12} < 1$, and only the two roots E_1 [1, 2] are suitable.

The FORTRAN program which gives the angular correlations (5) is easy to write taking account of all the conditions. This program is rapid and wastes the minimum of computer time to obtain the angular correlation.

2.2. *Decay into Two Unequal-Mass Particles*

Even when the masses of the emitted particles are different, we can write many relations identical to the preceding ones since they are obtained in the same way.

If the values E_R and E_R' are the resonance energies in the K and the K' systems, the relation between these energies is

$$E_R = \gamma E_R'. \tag{11}$$

The equations to solve, similar to equation (4) are

$$p_1^2 + p_2^2 + 2p_1p_2\cos\theta_{12} = \beta^2\gamma^2 E_R'$$

and

$$E_1 + E_2 = \gamma E_R'. \tag{12}$$

The angular distribution is given by the following relation:

$$\frac{d\sigma}{d\cos\theta_{12}} = \frac{E_R'}{2p_1'E_2'} \sum_\alpha \left| \frac{p_1^2[\beta^2\gamma^2 E_R' p_2 - \gamma E_2(p_2 + p_1\cos\theta_{12})]}{p_1p_2(E_1 - E_2) + \cos\theta_{12}(E_1p_2^2 - E_2p_1^2)} \right|_{E_1 = E_1^{(\alpha)}}. \tag{13}$$

The $E_1{}^{(\alpha)}$ values are the roots of the following E_1 equation:

$$2E_1{}^2 - 2\gamma E_R{}' E_1 - m_1{}^2 - m_2{}^2$$
$$= -2 \cos \theta_{12}[(E_1{}^2 - m_1{}^2)(E_1{}^2 - 2\gamma E_R{}' E_1 + \gamma^2 E_R'^2 - m_2{}^2)]^{1/2}. \tag{14}$$

It is impossible to derive the values $E_1^{(\alpha)}$ in a way like that of the previous paragraph because we have to solve quadratic equations. We wrote a FORMRAN program [3] which gives automatically the angular correlation (2) according to the roots of (14).

This program is less rapid than the preceding one, because we have to solve Eq. (14), which can be reduced to an algebraic equation in $E_1{}^4$. It is not recommended to use the general subroutine designed to get complex roots of a general equation. It is preferable, for a fast computation, to design a program that gives the real roots of an equation in x^4. We adopted this method to write our program. We have to note that the angular correlation is very sensitive to the accuracy of the roots of Eq. (14).

References

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